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Dichromatic squeezing generation

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Abstract. We propose a novel scheme for the joint generation of two squeezed beams at arbitrary frequencies ω_1 and ω_2. The scheme consists of two successive steps, both involving nonlinear interactions in $\chi^{(2)}$ crystals. The dynamics of the setup is analyzed both quantum mechanically and classically within the parametric approximation. An experimental implementation involving the fundamental and the harmonics of a Nd:YAG laser pulse, and $\beta{-}BaB_2O_4$ nonlinear crystals is suggested.

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1 Introduction

In the last two decades the phenomenon of squeezing has been extensively investigated both theoretically and experimentally. Besides the fundamental interest, squeezing is also a powerful resource for applications, since the manipulation of the quantum fluctuations allows to largely improve the efficiency of optical devices in quantum communication channels [1], atomic spectroscopy [2] and ultra-sensitive interferometric measurements [3]. In particular, the possibility of tuning the frequency is essential to improve sensitivity in spectroscopy and broadband interferometry [4].

Squeezed states are usually produced by degenerate parametric amplification, in which a signal mode at frequency ω is coupled with a pump mode at double frequency 2ω. The interaction is mediated by the second-order susceptibility tensor $\chi^{(2)}$ of the medium, such that each photon in the pump mode produces a photon pair in the signal mode. Therefore, in order to obtain squeezing at a desired frequency one needs a strong source at double frequency, and a suitable nonlinear crystal allowing phase-matching conditions for degenerate down-conversion. Schemes for tunable sources of squeezed light have been also reported using optical parametric oscillators operating below threshold [5], using diode-laser-based sources [6] and in the resonance fluorescence spectrum of a two-level atom [7].

In this paper we focus our attention on a novel scheme for the joint generation of two squeezed beams in two traveling waves at arbitrary frequencies ω_1 and ω_2. Such a dichromatic squeezing is obtained in two successive steps, both involving nonlinear interactions in $\chi^{(2)}$ crystals. In the first step, the nonlinear crystal is pumped at frequency $\omega_3 = \omega_1 + \omega_2$, in order to realize parametric amplification of the idler and signal modes. Starting from a couple of coherent beams, the output signal and idler modes are excited in a displaced twin-beam state at frequencies ω_1 and ω_2. In the second step, such an entangled state is injected into another $\chi^{(2)}$ crystal, which is pumped at frequency $\omega_4 = \omega_2 - \omega_1$, and works as a frequency converter between ω_2 and ω_1. The effective Hamiltonian describing the second step is equivalent, in the limit of a classical undepleted pump at ω_4 [8], to that of a balanced beam splitter, which mixes the signal and the idler modes. As a consequence of mixing, entanglement is transformed into squeezing [9,10], so that the output consists of a pair of uncorrelated squeezed beams at frequencies ω_1 and ω_2. Notice that the second crystal cannot be substituted by a real beam-splitter, since the two involved modes have different frequencies ω_1 and ω_2.

By suitably choosing ω_3 and ω_4 squeezing at any desired pair of frequencies can be achieved. In addition, the amplitudes and the squeezing of the output beams may be adjusted by suitably tuning the initial amplitudes and the gain of the amplifier respectively.

The paper is structured as follows. In Section 2 we give a quantum description of the overall system, showing that two squeezed beams appear at the output. In Section 3

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we present the classical approach to the parametric interaction inside the two nonlinear crystals. Although this approach cannot account for the genuine quantum properties of the output beams, it is worthwhile to perform a classical analysis with the aim of obtaining a reliable identification between classical and quantum parameters, in order to make some predictions on the experimental implementation of the system. In Section 4 we propose an experimental arrangement in order to implement the dichromatic squeezing generation at ω_1 and ω_2. The proposed setup is based on a pulsed Nd:YAG laser whose output is frequency doubled and tripled, and on two $\beta{-}BaB_2O_4$ (BBO) crystals. Finally, Section 5 closes the paper with some concluding remarks.

2 Quantum analysis

Our scheme consists of two successive steps, both involving $\chi^{(2)}$ nonlinear crystals. In the first step, the crystal operates as a nondegenerate optical parametric amplifier (NOPA), whereas in the second one it works as a frequency converter (FC). We start with two coherent beams at the input of the NOPA, and end up with a couple of uncorrelated squeezed beams at the output of the second crystal. In both steps we assume the validity of the parametric approximation, *i.e.* we consider the case of an undepleted coherent pump [8].

The dynamics of the NOPA involves three different modes of the radiation field. Hereafter, we denote these modes by a_1 the signal, a_2 the idler, and a_3 the pump. The modes, whose frequencies are linked by the relation $\omega_3 = \omega_1 + \omega_2$, are coupled by the medium nonlinearity. In the rotating-wave approximation, the Hamiltonian of the NOPA under phase-matching conditions can be written as follows

$$\hat{H}_1 \propto a_1 a_2 a_3^\dagger + a_1^\dagger a_2^\dagger a_3. \quad (1)$$

The interaction described by the Hamiltonian (1) covers a considerably rich variety of phenomena, such as generation of correlated photon pairs by parametric downconversion [11–13], phase insensitive amplification [14,15], and realization of Bell states [13,16,17]. The unitary evolution operator in the interaction picture reads

$$\hat{U}_\tau = \exp\left[-\mathrm{i}\tau\left(a_1 a_2 a_3^\dagger + a_1^\dagger a_2^\dagger a_3\right)\right], \quad (2)$$

where τ represents a rescaled interaction time. The parametric approximation consists in replacing the pump mode a_3 with the complex amplitude γ of the corresponding coherent state, thus achieving the two-mode squeezing operator

$$\hat{U}_\lambda = \exp\left[\lambda a_1^\dagger a_2^\dagger - \lambda^* a_1 a_2\right], \quad (3)$$

where the coupling λ is given by $\lambda = -\mathrm{i}\tau\gamma$. The two-mode squeezing operator yields a suppression of the quantum fluctuations in one quadrature of the sum and difference of modes $a_1 \pm a_2$ [18]. Let us consider two coherent beams at the input of the system $|\psi_0\rangle = |\alpha_1\rangle_1|\alpha_2\rangle_2 = \hat{D}_1(\alpha_1)\otimes\hat{D}_2(\alpha_2)|\mathbf{0}\rangle$, where $\hat{D}(\alpha) = \exp\left[\alpha a^\dagger - \alpha^* a\right]$ denotes the displacement operator, and $|\mathbf{0}\rangle$ the e.m. vacuum. The output from the first crystal is given by (we neglect an overall phase)

$$|\psi_1\rangle = \hat{D}_1(\mu\alpha_1 + \nu\alpha_2^*)\otimes\hat{D}_2(\mu\alpha_2 + \nu\alpha_1^*)\,|\psi_{\rm twb}\rangle \quad (4)$$

where $\mu = \cosh|\lambda|$, $\nu = \sinh|\lambda|$ (we assume, without loss of generality, a real pump amplitude γ) and $|\psi_{\rm twb}\rangle$ denotes the so-called twin-beam

$$|\psi_{\rm twb}\rangle = \hat{U}_\lambda\,|\mathbf{0}\rangle = \sqrt{1-|\chi|^2}\sum_{n=0}^{\infty}\chi^n\,|n\rangle_1\otimes|n\rangle_2, \quad (5)$$

which represents the maximally entangled state of the two modes a_1 and a_2. The expression in equation (5), with $\chi = \arg(\lambda)\tanh|\lambda|$, can be easily derived by factorizing $\hat{U}_\lambda$ through the decomposition formulas for the SU(1,1) Lie algebra [14,19,20], *i.e.*

$$\exp\left[\lambda a_1^\dagger a_2^\dagger - \lambda^* a_1 a_2\right] = \exp\left[a_1^\dagger a_2^\dagger\,\frac{\lambda}{|\lambda|}\tanh\lambda\right] \times\exp\left[-\log(\cosh^2\lambda)(a_1^\dagger a_1 + a_2^\dagger a_2 + 1)\right] \times\exp\left[-a_1 a_2\,\frac{\lambda^*}{|\lambda|}\tanh\lambda\right]. \quad (6)$$

The mode transformations of the NOPA are given by

$$\hat{U}_\lambda^\dagger\begin{pmatrix}a_1\\ a_2\end{pmatrix}\hat{U}_\lambda = \begin{pmatrix}\mu\,a_1 - \nu\,a_2^\dagger\\ -\nu\,a_1^\dagger + \mu\,a_2\end{pmatrix}. \quad (7)$$

The gain of the device can be defined by taking, as the initial condition, either the signal or the idler modes excited in some arbitrary state, and the other mode in the vacuum. By using equation (7) we arrive at the expression

$$G_j = \frac{\langle a_j^\dagger a_j\rangle_{\rm out} - \langle a_j^\dagger a_j\rangle_{\rm in}}{\langle a_j^\dagger a_j\rangle_{\rm in}} = \sinh^2|\lambda|\left(1 + \frac{1}{\langle a_j^\dagger a_j\rangle_{\rm in}}\right). \quad (8)$$

The first term in equation (8) coincides with the analogue quantity defined in the classical analysis of the parametric amplifier. On the other hand, the second term corresponds to the parametric downconversion of the vacuum (usually also referred to as parametric spontaneous emission), namely a genuine quantum effect. Indeed, the total number of photons carried by the twin-beam of equation (5) is given by

$$\langle\psi_{\rm twb}|a_1^\dagger a_1 + a_2^\dagger a_2|\psi_{\rm twb}\rangle = 2\sinh^2|\lambda|. \quad (9)$$

In the second step of our scheme the displaced twin-beam of equation (4) enters a second $\chi^{(2)}$ crystal, which is pumped by mode a_4 at frequency $\omega_4 = \omega_2 - \omega_1$. The Hamiltonian of the device, under phase-matching conditions and in the rotating wave approximation, can be written as follows

$$\hat{H}_2 \propto a_1 a_4 a_2^\dagger + a_1^\dagger a_4^\dagger a_2. \quad (10)$$

$$|\psi_2\rangle = \hat{V}_{\pi/4}|\psi_1\rangle = \underbrace{\hat{V}_{\pi/4}\hat{D}_1(\mu\alpha_1+\nu\alpha_2^*)\otimes\hat{D}_2(\mu\alpha_2+\nu\alpha_1^*)\hat{V}_{\pi/4}^\dagger}_{\boxed{1}}\;\underbrace{\hat{V}_{\pi/4}\,\hat{U}_\lambda\,\hat{V}_{\pi/4}^\dagger}_{\boxed{2}}\;\underbrace{\hat{V}_{\pi/4}|\mathbf{0}\rangle}_{\boxed{3}}, \tag{14}$$

We now consider the mode a_4 excited in a strong coherent state provided by a laser beam. Within the parametric approximation, the effective Hamiltonian of the second crystal reduces to

$$\hat{H}_2 \propto \chi^{(2)}(a_1^\dagger a_2 + a_1 a_2^\dagger), \tag{11}$$

and the corresponding unitary evolution operator is equivalent to that of a beam splitter [8], which by suitable rephasing of modes a_1 and a_2 reads

$$\hat{V}_\theta = \exp\left[-\mathrm{i}\theta\left(a_1 a_2^\dagger + a_1^\dagger a_2\right)\right]$$

$$\hat{V}_\theta^\dagger \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \hat{V}_\theta = \begin{pmatrix} \cos\theta\, a_1 + \sin\theta\, a_2 \\ -\sin\theta\, a_1 + \cos\theta\, a_2 \end{pmatrix}. \tag{12}$$

By an appropriate tuning of the effective interaction time (*i.e.* crystal length and pump intensity) the unitary evolution is equivalent to the action of a balanced 50/50 beam splitter on the modes a_1 and a_2

$$\hat{V}_{\pi/4} = \exp\left[-\mathrm{i}\frac{\pi}{4}\left(a_1 a_2^\dagger + a_1^\dagger a_2\right)\right]. \tag{13}$$

It is worth noticing that a true beam splitter cannot be used to implement the bilinear interaction between the modes a_1 and a_2 described by the Hamiltonian (11). In fact, a beam-splitter is a passive device, *i.e.* an optical medium where only the first-order susceptibility plays a role. Therefore, in a beam splitter a bilinear Hamiltonian of the form (11) can be realized only when the two involved modes have the same frequency. This is not the case of our system, where the relevant modes have different frequencies, and where the required beam-splitter-like Hamiltonian must be obtained as an effective Hamiltonian. Notice that our implementation is minimal, in that it uses only a second-order crystal and the minimum number of modes.

We also notice that our second stage is the reverse process of that implemented in reference [10], to obtain two entangled beams by mixing a couple of squeezed vacuum fields. In that case, however, the input modes were at the same frequency and therefore a simple "true" beam-splitter could be used.

In order to evaluate the action of $\hat{V}_{\pi/4}$ on the output from the first crystal, $|\psi_1\rangle$, we remind that $|\psi_{\mathrm{twb}}\rangle = \hat{U}_\lambda|\mathbf{0}\rangle$ and therefore

see equation (14) above

where we have inserted the identity operator $\hat{I} = \hat{V}_{\pi/4}^\dagger\hat{V}_{\pi/4}$ twice.

Equation (14) contains three terms, in order to simplify the expression we notice that

$$\boxed{3}\quad \hat{V}_{\pi/4}|0\rangle_1\otimes|0\rangle_2 = |0\rangle_1\otimes|0\rangle_2$$

$$\boxed{1}\quad \hat{V}_{\pi/4}\hat{D}_1(\mu\alpha_1+\nu\alpha_2^*)\otimes\hat{D}_2(\mu\alpha_2+\nu\alpha_1^*)\hat{V}_{\pi/4}^\dagger = \hat{D}_1(\gamma)\otimes\hat{D}_2(\delta) \tag{15}$$

(again an overall phase has been neglected) with

$$\gamma = \mu\frac{\alpha_1+\alpha_2}{\sqrt{2}} + \nu\frac{\alpha_1^*+\alpha_2^*}{\sqrt{2}}$$
$$\delta = \mu\frac{\alpha_1-\alpha_2}{\sqrt{2}} - \nu\frac{\alpha_1^*-\alpha_2^*}{\sqrt{2}} \tag{16}$$

and that [9]

$$\boxed{2}\quad \hat{V}_{\pi/4}\,\hat{U}_\lambda\,\hat{V}_{\pi/4}^\dagger = \exp\left[\frac{\lambda}{2}\left(a_1^{\dagger 2}-a_1^2\right) - \frac{\lambda}{2}\left(a_2^{\dagger 2}-a_2^2\right)\right] = S_1(\lambda)\otimes S_2(-\lambda), \tag{17}$$

which is the product of two "disentangled" squeezing operators $\hat{S}(\zeta) = \exp[\zeta/2(a^{\dagger 2}-a^2)]$ of the two modes.

The overall output state from the setup is thus given by

$$|\psi_2\rangle = \left[\hat{D}_1(\gamma)\otimes\hat{D}_2(\delta)\right]\left[\hat{S}_1(\lambda)\otimes\hat{S}_2(-\lambda)\right]|\mathbf{0}\rangle \tag{18}$$

i.e. a couple of squeezed state in the modes a_1 and a_2 at the frequencies ω_1 and ω_2. The squeezing amplitudes $|\lambda|$ is the same for the two modes, whereas the squeezing phases are shifted by π each other, which means that the two modes are squeezed in orthogonal directions. By varying the initial coherent amplitudes one may tune the final amplitudes according to equation (16). In particular, for vacuum input one has a couple of uncorrelated squeezed vacuum states at the output. On the other hand, the amount of squeezing S achievable at the output depends only on the gain of the parametric amplifier, *i.e.* on the strength λ of the nonlinear interaction in the first crystal. The squeezing S can be quantified through the reduction of the field fluctuations compared to the vacuum level $\langle\Delta x_\varphi^2\rangle_{\mathrm{vac}} = 1/4$, in formula

$$S = 4\min_\varphi\langle\Delta x_\varphi^2\rangle = \exp\big(-2|\lambda|\big). \tag{19}$$

In equation (19), $x_\varphi = 1/2(a\mathrm{e}^{-\mathrm{i}\varphi}+a^\dagger\mathrm{e}^{\mathrm{i}\varphi})$ denotes the field quadrature at phase φ; lower values of S corresponds to stronger squeezing. For vacuum input the squeezing of the two modes at the output is given by

$$S = (1+N) - \sqrt{N(N+2)}, \tag{20}$$

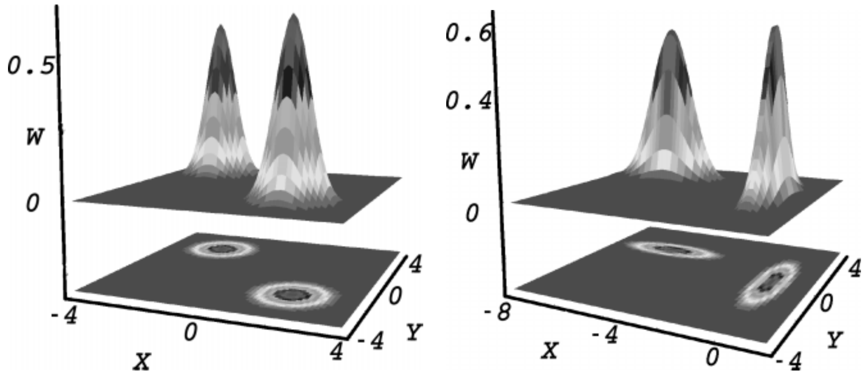

Fig. 1. Wigner functions of the input (left) and the output (right) states of the two beams traveling through the two-step scheme. The plots shows the Wigner functions for a couple of coherent states with initial amplitudes given by $\alpha_1 = (-2, 2)$, $\alpha_2 = (2, -2)$, and for a coupling $\lambda = 0.6$, such that the final amplitudes are $\gamma = (0, 0)$ and $\delta = (-5.15, 1.55)$, whereas the squeezing is given by $S = 0.3$

where N is the total number of photons carried by the twin-beam at the output of the first step (see Eq. (9)). For a couple of coherent states of amplitudes α_1 and α_2 at the input this expression is still valid with the substitution

$$N \to N^\star = \frac{2G_1|\alpha_1|^2}{1+|\alpha_1|^2} = \frac{2G_2|\alpha_2|^2}{1+|\alpha_2|^2},$$

G_1 and G_2 being the gain of the parametric amplifier evaluated on the signal and the idler mode respectively (see Eq. (8)).

We end this section with an example, to illustrate the dynamics of the proposed system. In order to obtain an immediate picture of the squeezing effects we describe the modes in terms of their Wigner functions. As the input, let us consider a couple of coherent beams of amplitudes $\alpha_1 = (x_{01}, y_{01})$ and $\alpha_2 = (x_{02}, y_{02})$ respectively (the vacuum is obtained as a special case with $x_{01} = y_{01} = x_{02} = y_{02} = 0$). The modes are uncorrelated and therefore the Wigner function $W^0(x_1, x_2, y_1, y_2) = W_1^0(x_1, y_1)W_2^0(x_2, y_2)$ is factorized in two terms that are given by

$$\begin{aligned} W_1^0(x_1, y_1) &= \frac{2}{\pi}\exp\left[-2(x_1 - x_{01})^2 - 2(y_1 - y_{01})^2\right] \\ W_2^0(x_2, y_2) &= \frac{2}{\pi}\exp\left[-2(x_2 - x_{02})^2 - 2(y_2 - y_{02})^2\right]. \end{aligned} \tag{21}$$

After the parametric amplifier the modes are entangled. The Wigner function corresponding to the displaced twin-beam of equation (4) is given by

$$\begin{aligned} W^1(x_1, x_2, y_1, y_2) = \\ W_{\rm twb}(x_1 - {\rm Re}[\mu\alpha_1 + \nu\alpha_2^*], x_2 - {\rm Re}[\mu\alpha_2 + \nu\alpha_1^*], \\ y_1 - {\rm Im}[\mu\alpha_1 + \nu\alpha_2^*], y_2 - {\rm Im}[\mu\alpha_2 + \nu\alpha_1^*]), \end{aligned} \tag{22}$$

where the Wigner function of the twin-beam state is given by

$$\begin{aligned} W_{\rm twb}(x_1, x_2, y_1, y_2) = \frac{4}{\pi^2}\exp\Big[- \frac{(x_1 - x_2)^2}{4\sigma_1^2} \\ - \frac{(x_1 + x_2)^2}{4\sigma_2^2} - \frac{(y_1 - y_2)^2}{4\sigma_2^2} - \frac{(y_1 + y_2)^2}{4\sigma_1^2}\Big] \end{aligned} \tag{23}$$

and $\sigma_1 = 1/2\exp(|\lambda|)$, $\sigma_2 = 1/2\exp(-|\lambda|)$.

After the second step the two modes are again uncorrelated, and the Wigner function factorized $W^2(x_1, x_2, y_1, y_2) = W_1^2(x_1, y_1)W_2^2(x_2, y_2)$. The two terms are given by

$$\begin{aligned} W_1^2(x_1, y_1) &= \frac{2}{\pi}\exp\left[-\frac{(x_1 - {\rm Re}[\gamma])^2}{2\sigma_1^2} - \frac{(y_1 - {\rm Im}[\gamma])^2}{2\sigma_2^2}\right] \\ W_2^2(x_2, y_2) &= \frac{2}{\pi}\exp\left[-\frac{(x_2 - {\rm Re}[\delta])^2}{2\sigma_2^2} - \frac{(y_2 - {\rm Im}[\delta])^2}{2\sigma_1^2}\right], \end{aligned} \tag{24}$$

where γ and δ are given by equation (16). Notice that the Gaussian character of the Wigner functions is conserved throughout the system. In Figure 1 we show the Wigner functions of the input and the output states for a particular set of parameters.

3 Classical analysis

The classical Maxwell equations for the collinear interaction among three plane waves at frequencies ω_1, ω_2 and ω_3 satisfying $\omega_3 = \omega_1 + \omega_2$ are [21]:

$$\begin{aligned} \frac{\rm d}{{\rm d}z}a_1 &= -{\rm i}ga_3a_2^*\exp(-{\rm i}\Delta kz) \\ \frac{\rm d}{{\rm d}z}a_2 &= -{\rm i}ga_3a_1^*\exp(-{\rm i}\Delta kz) \\ \frac{\rm d}{{\rm d}z}a_3 &= -{\rm i}ga_1a_2\exp({\rm i}\Delta kz), \end{aligned} \tag{25}$$

where a_j are the complex envelopes of the interacting fields, $\Delta k = k_3 - k_2 - k_1$ is the mismatch, z is the spatial coordinate inside the crystal and g is the coupling parameter in MKS units

$$g = d\sqrt{\frac{2\hbar\omega_1\omega_2\omega_3\eta_0^3}{n_1n_2n_3}}, \tag{26}$$

where η_0 is the vacuum impedance, n_j is the refraction index at frequency ω_j, and d is proportional to $\chi^{(2)}$ and dependent of the tuning angle.

The interaction in the first crystal, which is intended to produce a displaced twin-beam, requires the field a_3 to satisfy the parametric approximation, $a_3(z) = a_3(0)$. In this case system (25) reduces to:

$$\frac{\mathrm{d}}{\mathrm{d}z}a_1 = -\mathrm{i}ga_3(0)a_2^*\exp(-\mathrm{i}\Delta kz)$$
$$\frac{\mathrm{d}}{\mathrm{d}z}a_2 = -\mathrm{i}ga_3(0)a_1^*\exp(-\mathrm{i}\Delta kz). \qquad (27)$$

The general solution of system (27) is:

$$a_1(z) = \left[a_1(0)\cosh\left(\frac{\sqrt{|\gamma_3|^2-\Delta k^2}}{2}z\right) - \mathrm{i}\frac{\gamma_3 a_2^*(0)-\Delta k a_1(0)}{\sqrt{|\gamma_3|^2-\Delta k^2}}\sinh\left(\frac{\sqrt{|\gamma_3|^2-\Delta k^2}}{2}z\right)\right] \times \exp\left(-\mathrm{i}\frac{\Delta k}{2}z\right)$$

$$a_2(z) = \left[a_2(0)\cosh\left(\frac{\sqrt{|\gamma_3|^2-\Delta k^2}}{2}z\right) - \mathrm{i}\frac{\gamma_3 a_1^*(0)-\Delta k a_2(0)}{\sqrt{|\gamma_3|^2-\Delta k^2}}\sinh\left(\frac{\sqrt{|\gamma_3|^2-\Delta k^2}}{2}z\right)\right] \times \exp\left(-\mathrm{i}\frac{\Delta k}{2}z\right), \qquad (28)$$

where we defined $\gamma_3 = 2ga_3(0)$.

Since $|a_j(z)|^2$ represents the flux of photons (photons/(s m^2)), it is useful to define the gain factor of the process as:

$$\Gamma_j(z) = \frac{|a_j(z)|^2 - |a_j(0)|^2}{|a_j(0)|^2}, \qquad (29)$$

being $j = 1, 2$. From (28) in the case of perfect phase-matching ($\Delta k = 0$) we get:

$$\Gamma_1(z) = \frac{|a_1(0)|^2+|a_2(0)|^2}{|a_1(0)|^2}\sinh^2\left(\frac{|\gamma_3|}{2}z\right) - \frac{|a_2(0)|}{|a_1(0)|}\sin\left(\Lambda_1(0)+\Lambda_2(0)-\Lambda_3(0)\right)\sinh\left(|\gamma_3|z\right)$$

$$\Gamma_2(z) = \frac{|a_1(0)|^2+|a_2(0)|^2}{|a_2(0)|^2}\sinh^2\left(\frac{|\gamma_3|}{2}z\right) - \frac{|a_1(0)|}{|a_2(0)|}\sin\left(\Lambda_1(0)+\Lambda_2(0)-\Lambda_3(0)\right)\sinh\left(|\gamma_3|z\right), \qquad (30)$$

where $\Lambda_j(0)$ are the initial phases of the $a_j(0)$. We note that there is an optimum choice for these initial phases, namely: $\Lambda_1(0)+\Lambda_2(0)-\Lambda_3(0) = -\pi/2$, which maximizes the gain value of both the frequency components in the twin-beam:

$$\Gamma_1(z) = \frac{|a_1(0)|^2+|a_2(0)|^2}{|a_1(0)|^2}\sinh^2\left(\frac{|\gamma_3|}{2}z\right) + \frac{|a_2(0)|}{|a_1(0)|}\sinh\left(|\gamma_3|z\right)$$
$$\Gamma_2(z) = \frac{|a_1(0)|^2+|a_2(0)|^2}{|a_2(0)|^2}\sinh^2\left(\frac{|\gamma_3|}{2}z\right) + \frac{|a_1(0)|}{|a_2(0)|}\sinh\left(|\gamma_3|z\right). \qquad (31)$$

The above expression allows us to find a relation between the quantum coupling parameter λ and the classical one g. In fact, if we consider the classical limit to the quantum equations describing the twin-beam generation, the gain in the number of photons starting from a non-vacuum initial state for one of the two beams is constituted by the first term in equation (8). If we evaluate the classical gain at the exit of the first crystal (Eq. (30)) in the same conditions and for perfect phase-matching we get, for the initially non-zero field:

$$\Gamma_j = \sinh^2\left(\frac{|\gamma_3|}{2}z\right). \qquad (32)$$

By identifying the gain in equation (32) with the first term in equation (8), we obtain:

$$|\lambda| = \frac{|\gamma_3|}{2}z = g|a_3(0)|z, \qquad (33)$$

that is the required relation between quantum and classical parameters. Thus we can use this identification as a parameter for choosing a suitable second-order non-linear crystal and the type of interaction that allows the generation of the desired number of coupled photons.

In the second step of the process, by which we want to produce two disentangled squeezed states, the fields exiting the first crystal of thickness L are used as input fields for a three wave interaction with a field $\bar{a}_4$, whose intensity allows the parametric approximation, $\bar{a}_4(z) = \bar{a}_4(0)$, and whose frequency satisfies $\omega_4 = \omega_2 - \omega_1$. As initial values of fields $\bar{a}_j$ ($j = 1, 2$) we can take $\bar{a}_j(0) = |a_j(L)|\exp(\mathrm{i}\bar{\Lambda}_j(0))$. System (25) reduces to:

$$\frac{\mathrm{d}}{\mathrm{d}z}\bar{a}_1 = -\mathrm{i}g\bar{a}_2\bar{a}_4^*(0)\exp(-\mathrm{i}\Delta kz)$$
$$\frac{\mathrm{d}}{\mathrm{d}z}\bar{a}_2 = -\mathrm{i}g\bar{a}_1\bar{a}_4(0)\exp(\mathrm{i}\Delta kz). \qquad (34)$$

and its general solution becomes:

$$\bar{a}_1(z) = \left[\bar{a}_1(0)\cos\left(\frac{\sqrt{|\gamma_4|^2+\Delta k^2}}{2}z\right) - \mathrm{i}\frac{\gamma_4^*\bar{a}_2(0)-\Delta k\bar{a}_1(0)}{\sqrt{|\gamma_4|^2+\Delta k^2}}\sin\left(\frac{\sqrt{|\gamma_4|^2+\Delta k^2}}{2}z\right)\right] \times \exp\left(-\mathrm{i}\frac{\Delta k}{2}z\right)$$

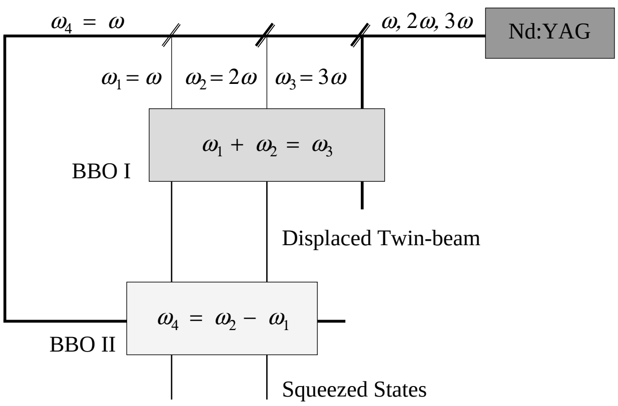

Fig. 2. Block diagram of the experimental setup for the generation of squeezed states at fundamental and second harmonic frequency.

$$\bar{a}_2(z) = \left[\bar{a}_2(0)\cos\left(\frac{\sqrt{|\gamma_4|^2+\Delta k^2}}{2}z\right) - \mathrm{i}\frac{\gamma_4\bar{a}_1(0)+\Delta k\bar{a}_2(0)}{\sqrt{|\gamma_4|^2+\Delta k^2}}\sin\left(\frac{\sqrt{|\gamma_4|^2+\Delta k^2}}{2}z\right)\right] \times \exp\left(+\mathrm{i}\frac{\Delta k}{2}z\right). \quad (35)$$

where we defined $\gamma_4 = 2g\bar{a}_4(0)$ and $\Delta k = k_2 - k_4 - k_1$.

By the use of the same definition of above for the gain of the process, we get for perfect phase-matching ($\Delta k = 0$) and $|\bar{a}_1(0)| = |\bar{a}_2(0)|$ (which is true when fields 1 and 2 were generated as a twin-beam):

$$\bar{\Gamma}_1(z) = -\sin\left(\bar{\Lambda}_1(0)+\bar{\Lambda}_4(0)-\bar{\Lambda}_2(0)\right)\sin\left(|\gamma_4|z\right) = -\bar{\Gamma}_2(z), \quad (36)$$

where $\bar{\Lambda}_j(0)$ are the phases of the fields $\bar{a}_j(0)$ at the input of the second crystal.

The identification between classical and quantum parameters in this second step can be easily carried out by comparing the coefficients of $\bar{a}_1(0)$ and $\bar{a}_3(0)$ in equations (35), in the case of phase matching, with those of the field transformation produced by a beam splitter (see Eq. (12)). We have

$$|\gamma_4|z = 2g|\bar{a}_4(0)|z = \frac{\pi}{2}. \quad (37)$$

This identification can be used to design an experimental arrangement producing dichromatic squeezing.

4 Experimental proposal

An experimental implementation of the scheme proposed in this paper can be obtained by pumping the first and the second crystal with the third harmonics and the fundamental of a laser pulse respectively (see Fig. 2). If the first crystal is tuned for generating the displaced twin-beam at the second-harmonic and at the fundamental wavelengths *via* parametric down-conversion, the output disentangled squeezed states will also be at the fundamental and at the second harmonic wavelengths.

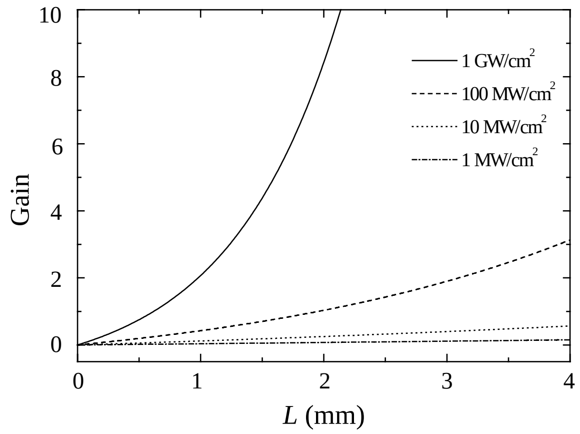

Fig. 3. Values of the gain in the first step (Γ_j in Eq. (31) for $|a_1(0)| = |a_2(0)|$) as a function of the crystal thickness at different intensities of the pump at ω_3 for a BBO crystal in phase-matching I. The interaction parameters are: tuning angle $\alpha = 31.3°$, coupling $g = 1.32748 \times 10^{-13}$.

The neodymium laser has harmonics readily available up to the fourth. If, as we propose, a Nd:YAG laser is chosen, the wavelengths involved in the process will be: $\lambda_1 = 1064$ nm, $\lambda_2 = 532$ nm, $\lambda_3 = 355$ nm and $\lambda_4 = 1064$ nm. Concerning the choice of the nonlinear crystals to be used in the two steps of the process, they must satisfy phase-matching conditions and maximize the efficiency of the nonlinear interaction at the given frequencies, *i.e.* maximize the efficiency of the twin-beam generation. For our purposes, and at the given wavelengths, the most suitable crystal is the BBO [23–25].

The first step can be realized through a type-I phase-matching interaction with an extraordinarily-polarized pump. In this way, the twin-beam photons are both ordinarily polarized. In Figure 3 we show the values of gain (number of generated photons at ω_1 and ω_2) at the end of the first crystal (Γ_j in Eq. (32)) as a function of the BBO crystal thickness for different input intensities ($I_3 \propto |\gamma_3|^2 \propto |a_3(0)|^2$) for this type-I interaction. Within the parametric approximation, the coupling of the frequency conversion occurring in the second crystal is a periodic function (see Eq. (36)) with a period linearly dependent on both the intensity of the pump ($I_4 \propto |\gamma_4|^2 \propto |a_4(0)|^2$) and the crystal thickness, z. When the interaction parameters are chosen so that an effective balanced beam-splitter-like Hamiltonian is obtained (see Eq. (37)), the coupling will depend on the relative initial phases only. In order to implement the second step we propose to realize a type-II phase-matching interaction in which the wave at ω_1 has ordinary polarization while those at ω_2 and ω_4 have extraordinary polarization. This can be obtained by positioning a $\lambda/4$ plate on the beam at ω_2 so as to rotate the polarization of the photons at ω_2 but not that of the photons at ω_1. In Figure 4 we show the expected squeezing level of the output states

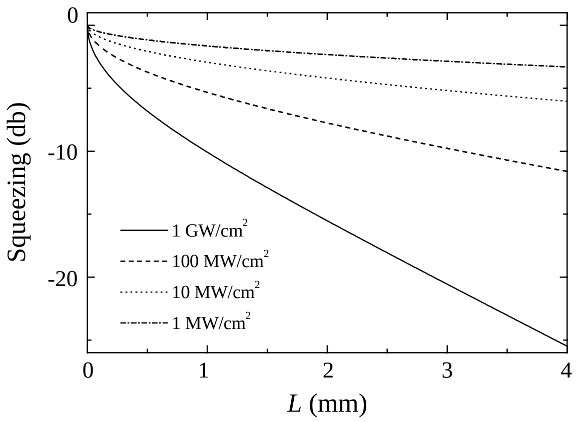

Fig. 4. Squeezing of the modes at the output of the system, as a function of the first crystal thickness, for the gain values as in Figure 3.

(as a function of the thickness of the first crystal) for the same parameters as in Figure 3 calculated as $10 \log S$ where S is given by equation (20).

A tunable source of dichromatic squeezed states can be experimentally obtained by using a laser source able to produce a tunable frequency ω_4 in the range of all the differences between frequencies ω_1 and ω_2 that can be phase matched with ω_3 by tuning the first crystal in angle. This can be achieved through a Raman cell pumped by the fundamental or the frequency-doubled output of a Nd:YAG laser. By varying the material in the Raman cell we can vary the Raman-shift and hence frequency ω_4.

5 Conclusions

In this paper we have suggested a novel two-step scheme to jointly generate squeezing at two arbitrary frequencies. The scheme involves nonlinear $\chi^{(2)}$ crystals and is feasible with current technology. The output beams are uncorrelated and both are excited in a squeezed state, whose degree of squeezing depends on the amplifier gain in the first step of the device. The tunability of the dichromatic output depends on the ability to vary the frequency ω_4 at which the second crystal is pumped in the range of all possible differences between the frequencies ω_1 and ω_2 covered by the first-crystal tuning range. This possibility is offered by a Raman cell pumped by the fundamental or the frequency doubled Nd:YAG laser output, in which the Raman material is chosen in order to adjust the Raman-shift. A direct implementation of our setup (without the need of Raman cell) can be obtained using the fundamental, second and third harmonics of a Nd:YAG pulsed laser, interacting in two BBO nonlinear crystals.

Concerning the detection of the squeezed states generated by our system, we note that the proposed realization of the scheme *via* Nd:YAG harmonics allows a straightforward implementation of the homodyne detection technique on both output beams, since the required local oscillators (at fundamental and at second harmonics) can be directly extracted from the laser source. On the other hand, when ω_4 is generated through Raman shift in order to reveal dichromatic squeezing further processes of sum- and difference-frequency conversion are required to produce the local oscillator at the appropriate frequency.

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